GQ(2, 4), Split Cayley Hexagon of Order Two and Black Hole Entropy Formulas

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Abstract:
It will be shown that the $E_{6(6)}$ symmetric entropy formula describing black holes and black strings in D=5 is intimately tied to the geometry of the generalized quadrangle GQ(2, 4) [1,2]. The 27 charges correspond to the points and the 45 terms in the entropy formula to the lines of GQ(2,4). Different truncations with 15, 11 and 9 charges are represented by three distinguished subconfigurations of GQ(2,4), very well-known to finite geometers; these are the "doily" (i. e. GQ(2, 2)), the "perp-set" of a point, and the "grid" (i. e. GQ(2, 1)), respectively. In order to obtain the correct signs for the terms in the entropy formula, we employ a non-commutative labelling for the points of GQ(2, 4). For the 40 different possible truncations with 9 charges this labelling yields 120 so-called Mermin squares – objects well known from studies concerning Bell-Kochen-Specker-like theorems. These results are connected to our previous ones [3] obtained for the $E_{7(7)}$ symmetric entropy formula in D=4 by observing that the structure of GQ(2,4) is linked to a particular kind of geometric hyperplane of the split Cayley hexagon of order two, featuring 27 points located on 9 pairwise disjoint lines at maximum distance from each other (a distance-3-spread).

1. Lévay, P., Saniga, M., Vrana, P., and Pracna, P.: 2009, Black Hole Entropy and Finite Geometry, Physical Review D, Vol. 79, No. 8, 084036 (12 pages); arXiv:0903.0541.
2. Saniga, M., Lévay, P., Pracna, P., and Vrana, P.: 2009, The Veldkamp Space of GQ(2,4); arXiv:0903.0715.
3. Lévay, P., Saniga, M., and Vrana, P.: 2008, Three-Qubit Operators, the Split Cayley Hexagon of Order Two and Black Holes, Physical Review D, Vol. 78, No. 12, 124022 (16 pages); arXiv:0808.3849.